Multivariate statistics and archaeology

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THE ARCHAEOLOGICAL RESEARCH PROCESS

European archaeology has always been considered to be a humanistic discipline, with all its sympathetic insight into the life of the human beings with which it deals. Yet the archaeological data material has no humanistic touch - a collection of dead items bound together by context information. It can be qualified according to context, and the logic of the contexts can be outlined, but it has no inherent humanistic content that can be read as one reads a book.

This disparity in quality between the aim and the means of attaining this aim very often leads to schizophrenic studies in archaeology. On the one hand, we find formalised analyses of artifacts and context information without the slightest reference to 'the Indian behind the artefact', and on the other, far-reaching tales are spun, often without a sound base in the data material. For most archaeologists, this schizophrenia is neatly organised. In one chapter they painstakingly deal with the artifacts and their setting. They describe, measure, compare and summarise, with or without the help of statistics. Then, in the next chapter they discuss and draw conclusions in historical terms about extinct human societies.

Sometimes the schizophrenia is so perfect that hardly any of the painstaking analyses are used for any purpose at all in the concluding chapter. One cannot help wondering what all the descriptions were for.

Some archaeologists do not suffer notably from this disease. They either simply discard the conclusions in terms of human society, and lose themselves in the rigoristic world of 'stamp collecting', or they completely forget about the archaeological record, and become in Flannery's words 'born-again philosopher' (1982).

It is tempting to speak ironically of this unhappy state of affairs. However, it does conceal a very serious problem. What is the nature of the link between the archaeological record and the interpretations in terms of human culture and history?

The archaeological research situation involves two very different realities. One is the reality of prehistoric societies. This is a reality that can no longer be observed even though it is the target of archaeological research. The other is our present reality, which we can observe. The archaeological record is an integrated part of this present reality, and it remains part of this reality no matter how intensely we observe it. We cannot observe the past. There is no logical link that takes us from the archaeological record back into the past, and there is no way we can draw conclusions by rules of logic from the archaeological record to the nature and organisation of past societies.

Yet the archaeological record is real, and basically part of a past reality. This means that there are logical links from the past to the archaeological record, and if only we knew the past, and knew the nature of all the successive transformations that shaped the archaeological record (Schiffer 1972, 1976), we could predict it in great detail. Thus we can state that the archaeological record is structured by the past even if it is part of the present, and consequently there must be a correspondence in structure between the two. This we may use as a guiding principle to evaluate propositions concerning the past, and indeed it is the only link we have to past reality.

Initially, we may separate two obvious levels on which archaeologists work. One is the level of current reality, where we can observe, analyse and categorise with great precision. The other is the imaginary level of the past to which we ascribe qualities and causal relationships. The latter is as much a part of our current reality as the former, and the justification for claiming that our modelling on this level has relevance for the past depends on our ability to show that the structure of the propositions we put forward do not violate the structure of the archaeological record. It is worth noting that we can never prove a statement concerning the nature and organisation of past societies to be true beyond doubt. In simple cases we may feel very certain that our statements are right, even to the degree where we may be tempted to claim that we have drawn a logical conclusion from the archaeological record. With more complex models and general explanations, we can never claim to be certain, and in my opinion these general statements are in fact more a revelation of our current views upon present world realities, than they are statements of facts concerning past realities.

One important point should not be forgotten here. Although we cannot prove anything to be true, we certainly can falsify statements concerning the past. In theory, at least, we can outline the implications of a statement and compare these implications with the actual archaeological record. In simple matters this works quite well. In connection with complex statements, however, one may seriously doubt our ability to draw the right conclusions.

As mentioned, the two levels on which archaeology has to operate can be seen to be pursued independently by many scholars, and most archaeologists have a tendency to keep them separate in their works. However, only when we exploit the two levels simultaneously and try to maintain a strict correspondence between them can we make sensible progress. This means that, all through dealing with the archaeological record, we should keep our picture of the past and its implications in mind, and all through forming and altering our picture of the past, we should be acutely aware of the realities of the archaeological record. Whenever we acknowledge that the implications of our models for the past do not fit the data at hand, we should modify or completely discard our models. All along we must realise that we never work in a vacuum; we are always guided by preconceived ideas. Thus models and ideas come prior to data, but at the same time our picture of the past has credibility only, when it is not refuted by the archaeological record.

In European archaeology the actual approach to the archaeological research process has always been dominated by traditional positivism. Seemingly, within this approach there are no problems at all concerning the linking process. Knowledge of the past is believed to be a direct additive outcome of information extracted from the archaeological record (Childe 1956; Malmer 1984). If only we gather enough information, and if only we analyse the information thoroughly, we will have all we need for an understanding of prehistoric society. Unfortunately, there is no obvious solution to the problem that arises when two scholars working in general with the same material come to two different views of past societies. As they both add together the same figures, but reach different results, one of them must be wrong. An evaluation of the professional standing of the two adversaries seems to be the only way out (Thompson 1956).

Positivism in this version may apparently work well as long as everything, including the analysis of the archaeological record, is done intuitively. Then nobody can follow the steps in the research procedure. But as soon as formalised data analysis is adopted, problems arise. The obscurity of the linking between analyses and syntheses becomes evident, and as the analyses become more and more technical, the crack widens to a gap. Indeed, the schizophrenic behaviour of the archaeologist becomes painfully clear in the publications.

I certainly hold it true that formalised artifact analysis using various forms of 'exact' descriptions and statistics (McBurney 1967; Malmer 1962; Cullberg 1968 among others), has never increased our knowledge of prehistoric societies one bit more than less formal studies have, but it does indeed create voluminous and unreadable books. Often it is the formal analyses and statistics that get the blame, and heated reactions against this 'technological Frankenstein's monster' can be seen (Hawkes 1968:262). It is not generally realised that it is the approach itself that is wrong.

The hypothetical-deductive method introduced from American archaeology in the sixties and seventies never had any notable impact on European archaeology. This is in some ways sad, because its demands for an explicit linking between the hypotheses concerning prehistoric societies and the realities of the archaeological record give it operational strength, and remove (in theory at least) the possibility of excessive, aimless analysis of data materials. It was, however, presented in the archaeological literature (Fritz & Plog 1970; Watson et al. 1971) as an inseparable part of the deductive-nomological model of explanation, and it was really this model that for various reasons did not suit European archaeologists.

The Hemplian approach to explanation is quickly dying in anthropological research today, if for no other reason than for its lack of ability to produce anything but 'Mickey Mouse' laws of culture (Eggert 1982:141, with reference to Flannery). Hopefully, this may free the hypothetical-deductive method from its association with nomological positivism, and give it a less rigid appearance than it required in that company.

The adoption of a hypothetical-deductive approach has two advantages. It forces the linking process between data and synthesis to be transparent, and it gives a more dynamic goal-orientated exploitation of the data material. It does not, however, give the linking itself greater security, as one might be tempted to believe from the writings of Watson et al. (1971).

The only way that the linkage can be made more precise and secure is through the study of what has been termed archaeological formation processes (Schiffer 1976) or more grandiosely 'middle range research' (Binford 1983). By studying how an archaeological record is formed in present-day context, and how in general terms various elements of a living society influence this record, a better understanding can be reached of how a true archaeological record with roots in the past might have been created, and what this indicates in terms of a living society.

The study of archaeology, then, consists of three separate levels, which can be pursued individually, but should preferably not be. One is the theoretical level, where mentally modelled reconstructions of prehistoric societies are made, where cultural relations are specified, and where cultural changes are explained. This level logically has precedence over the others, but it cannot exist meaningfully unless it is constantly linked to the archaeological record.

The second level is the linking process. It consists of statements and arguments of how the archaeological record was formed, with direct reference to a conscious model of the past society in question. The logic of this process runs from the model to the archaeological record. Yet, it is not merely an intellectual exercise, as one might believe from deductive positivism. Empirical knowledge can and should indeed enter the linking arguments. Such a knowledge has definitely always been a part of the linking process introduced through the 'life experience' of archaeologists. However, realising the nature of the linkage it is far more profitable to rely on a formal study of present day formations of the 'archaeological record'. Because of the empirical content of the linking argumentation, the linkage itself seldom appears as a deduction from our models, and there is no reason why it should. The linking argumentation may take any form we wish, as long as we are aware that we cannot make a link unless we have theories and models concerning the prehistoric past; that the logic proceeds from these to the present day context; and consequently that it is wasteful not to explicate theories and models in advance of an attempt to link.

The third level concerns the factual study of the archaeological record in all its many-sided aspects. This is the level where archaeologists really feel at home, and the methods and techniques of this level have been developed to a high degree of perfection. The fun and pleasure of working at this level often make archaeologists forget that it is absolutely futile to work with the archaeological record without an ever present awareness of its relevance to the level of actual theories, models and reconstructions.

STATISTICS AND ARCHAEOLOGICAL RESEARCH

It is an obvious and legitimate question to ask: what has the preceding chapter to do with the use of statistics in archaeology? The present chapter tries to answer this question in some detail, and hopefully it will straighten out some misconceptions concerning statistics as well as place statistics in a more useful framework of application than has so far been the case in Scandinavian archaeology.

To begin with, I will cite Spiegel's account of the difference between inductive and deductive statistics.

> If a sample is representative of a population, important conclusions about the population can often be inferred from analysis of the sample. The phase under which such inference is valid is called inductive statistics or statistical inference. Because such inference cannot be absolutely certain, the language of probability is often used in stating conclusions. The phase of statistics which seeks only to describe and analyse a given group without drawing any conclusions or inferences about a larger group, is called descriptive or deductive statistics (1972:1).

Inductive approaches include probability estimation methods based on various theoretical distributions like the binomial, normal and Poisson distributions as well as statistical decisions based on various tests like the well known Chi-square and Student's t tests. It also includes inferences using various forms of regression analysis.

Deductive approaches include all types of descriptive statistics from various simple graphic and arithmetic descriptions of individual variables - alone or two by two - to the complicated multivariate data-reducing analyses which are the main issue of this book.

I would argue that the application of inductive statistics in archaeological research is very problematic. There are two main reasons for this, both of which stem from the nature of the archaeological record. Most inductive statistics require that we know in detail the distributional qualities of the populations to which we apply the inferences. At the same time, they require that we have complete control of the formation of the samples from which we infer. None of these requirements are met in archaeology. Whether we conceive of the 'populations' as a material present in the past, or just as a material present in the earth today, we have to realise that the populations of archaeological material and their distributional qualities are basically unknown. Furthermore, if we speak of past populations, we have no way of knowing the exact history of the formation of the samples. The same, of course, does not necessarily apply if we speak of populations in terms of the hidden part of the archaeological record.

A more fundamental objection against inductive statistics in archaeology may, however, be raised. It is very doubtful whether the archaeological record can be considered to be a sample of anything at all in a statistical sense. That is, the archaeological finds and their compositions cannot be viewed as samples that reveal what some larger background unit looked like. Each find - and each composition - springs from an actual historical event or sequence of events, and thus has a complete 'as it is' meaning by itself. It is a unit of complete information. Even if we speak of excavation samples from, say, a large settlement site, we cannot claim that these are samples in a statistical sense, for the settlement site itself is not a population with some uniform theoretical structure. On the contrary, it is a statistically very haphazard phenomenon, and predicting what the rest will be like from an excavation 'sample' in one part of the settlement is beyond statistics. It is of course possible to devise a sampling strategy that will reveal the structure of the settlement, but then we are not dealing with one, but many samples, and the statistics needed to reveal their information are not inductive, but deductive.

In sociology, there are no problems using inductive statistics, because it is possible to observe the populations and their qualities, and carefully define the extraction of samples in a way that makes it possible to use inference statistics in a meaningful way. As outlined above, the nature of archaeological research does not allow us to observe the original populations from which the archaeological record is extracted, nor can we follow - let alone define - the processes through which the record is extracted. Therefore, a use of inductive statistics in archaeology will for theoretical reasons be a misapplication, and I fear that in most cases it will also in practice lead to erroneous, or rather, nonsensical results.

Deductive statistics, in general, have no a priori assumptions that we cannot control. The specific methods do have limitations that impose restrictions on the data they analyse. However, these restrictions always apply directly to the observed input data. This leaves us in full control to use the methods properly.

It is very important to stress that deductive statistics are descriptive. They have no inferential value whatsoever. We can use them to clarify the contents of the archaeological record, and present it in a form that makes it easier for us to carry out our linking argumentation. They cannot in any way produce the conclusions for us.

Deductive statistics may be used on several levels. We can describe and summarise individual variables, we can describe the relationships between pairs of variables, and we can describe the structure of the interrelations of many variables. Whatever we choose to do, we should never forget to do it with a specific purpose in mind. It is easy to fill page upon page with descriptions using uni-variate and bi-variate statistics, but if we do not intend to use it in the linking argumentation, then why waste time, effort, and expensive pages?

Special problems relate to the methods that deal with many variables simultaneously. Today, the multivariate methods are easy to carry out at a technical level, but they are not so easy to understand as the uni-variate and bi-variate methods. Furthermore, they are easy to misapply if not understood correctly, and if some basic rules are not observed.

The main problem with multivariate analysis lies with the rather complicated treatment that input data are given. If it is not correctly understood what goes on between input and output in these analyses, there is an immediate danger that the output will be used as data for the linking argumentation on false premises.

A further problem is that whereas uni- and bi-variate methods in general gives straightforward pictures of the material they analyse, the same is not necessarily true with multivariate analyses. Indeed it is not uncommon to see analyses of genuine data that give very unsatisfactory or even misleading results. It is not sufficient to disclaim it with a 'garbage in, garbage out' shrug, as garbage very often comes out of obviously good archaeological data.

There are two points that I should like to emphasise in this connection. One is that multivariate analyses themselves are not atheoretical. In order to cope with a multivariate situation and represent it in a low-dimensional sub-space, the methods are by nature data-reductive. They subtract the 'unimportant' and leave only the 'important' information for further analysis and interpretation. The principles laid down in the methods to separate important from unimportant and to decide just how comparisons are to be made constitute the essence of the methods. If these principles do not fit the ideas guiding the data selection for an analysis, or the idea of what is relevant in the data material, then it may indeed be very difficult to obtain a reasonable result.

The second point to emphasise is that our ways of thinking in terms of the research process are extremely important here. Were we to adhere to an inductive positivist notion and consequently have a strategy where we collect and describe a lot of material, and then feed everything into a multivariate analytical method, then we would be bound to get a lot of nonsense out. No matter how sophisticated methods are, or may become, they will never be able to make a judgement of relevance between the individual variables. A judgement of relevance has to be made before analysis starts, it has to continue throughout the analyses, and it is entirely the responsibility of the archaeologist. To work with multivariate analysis and obtain good results in the long run means that: you have to define carefully your problem and your intended solution to this problem; you have to stipulate which variables are relevant to the solution, and only carry out analyses of those variables; and finally you should never stop with the first analysis - you should continuously question your preconceived models and the relevance of the data chosen, until you reach a model and a result from the analyses that can safely be linked together. That is definitely the way to do successful research using multivariate statistics. It is also in my opinion the factual practice of archaeological research in general, as outlined in the first section.

In the following I shall look into a few of the multivariate statistical methods and outline the principles by which they work, the areas of possible application within archaeology, and first and foremost the nature and quality of the results they give.

AN OUTLINE OF THREE 'FACTOR-ANALYTICAL' METHODS

Multivariate data may be treated in many different ways. There is an almost infinite choice of techniques, from the very simple to the very complex, that may be applied. Many of these techniques have proved useful to archaeology, and an overall archaeological evaluation of these methods could easily fill an entire book.

In this chapter I shall be concerned with three methods only, all found within the broad and very heterogeneous category called factor analysis. The three methods, principal component analysis (PCA), principal coordinate analysis (PCO) and correspondence analysis (CA), are closely related, since they share the same basic computational principles. Together they form a group of analyses that can handle all types of variables that one may possibly think of in any archaeological material. As they are also the methods used in most of the papers in this book, there is a good reason to discuss their application in archaeology in some detail.

All three methods make use of orthogonal regression applied to a point scatter in a multivariate metric space. As the methods take different types of variables as their input, their ways of reaching a representation in a metric space differ considerably, and consequently, the results from the three analyses are not entirely comparable. They have different qualities, even though the basic computational method is the same.

As not all readers of this book will be familiar with the principles of orthogonal regression in a multidimensional space, let alone its actual computation, a brief outline here may be helpful. Although an understanding cannot be obtained without introducing rather complex mathematics, the use of formulas will be avoided here on purpose, since it would probably scare off more readers than it will enlighten.

First of all, what do we understand by a metric representation in a multivariate space? Consider first a variable that measures some property. It could be a measure of length on a set of comparable items. This would be a measure on a metric scale, and it could be depicted as a scatter of points along an axis (Figure 1). Another variable could measure another property, say width, on the same set of items. This would also be a measure on a metric scale, and we could now depict our two variables in a two-dimensional coordinate system on the paper (Figure 2). If we added yet another variable measured on a metric scale, we could depict the three variables together in a solid-state three-dimensional model, but it would be difficult to make an acceptable representation on a piece of paper. Should we add a fourth, fifth or any number of variables measured on a metric scale, then we are beyond any geometric representation. Yet, arithmetically we may still speak of a metric representation, where each item is represented by one point in a space with a number of dimensions that equals the number of metric variables used to describe the item. Exactly the same rules of metric distance between the items apply in such a multivariate space, as in a normal three-dimensional space.

In order to understand the idea of an orthogonal regression applied to points in such a multivariate metric space, we will have to return to the two-dimensional case. To express the relationship between the two variables of the scattergram Figure 2, we would normally place a regression line through the scatter. This would either represent 'width' expressed as a function of 'length', or 'length' as a function of 'width'. In the first case the regression line would be found by minimizing the sum of squared vertical distances from the points to the line we are seeking. In the latter case the regression line would be found by minimizing the sum of squared distances horizontally from the points to the line. The two lines obtained would not be identical, as they are determined by the variable chosen as the independent.

As we have no interest in giving primacy to a specific variable, the ordinary regression method cannot be used here to express the relationship between the two variables. We need a method that is 'neutral' with respect to the variables, and determined by the scatter of points only. Such a line can be found by minimizing the sum of squared distances perpendicular from the points to the line we are seeking. This line is known as the orthogonal regression line. It will pass through the points in the graph that represent the mutual mean of the variables (the origo), and it is therefore preferable to scale the variables so that they have zero as their mean value (Figure 3).

Figure 1. A graphical representation of one metric variable.

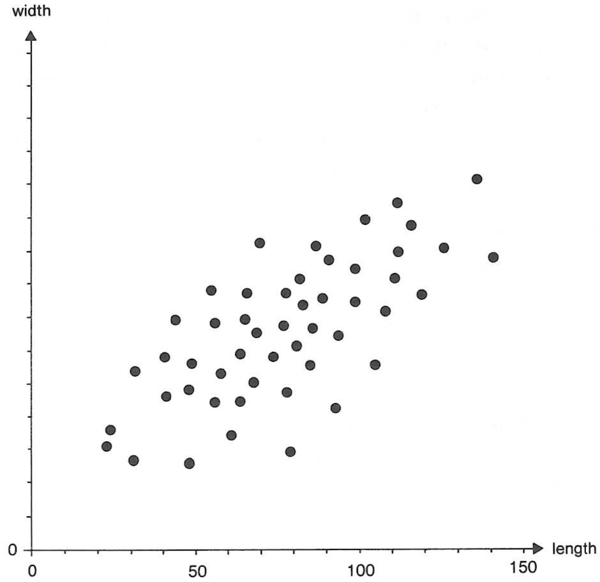

Figure 2. A graphical representation of two metric variables.

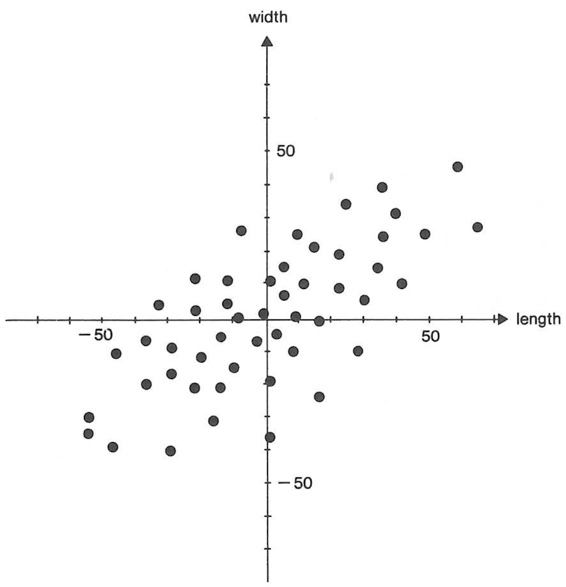

Figure 3. The same data as in Figure 2, but with the two variables scaled to have zero as their mean value.

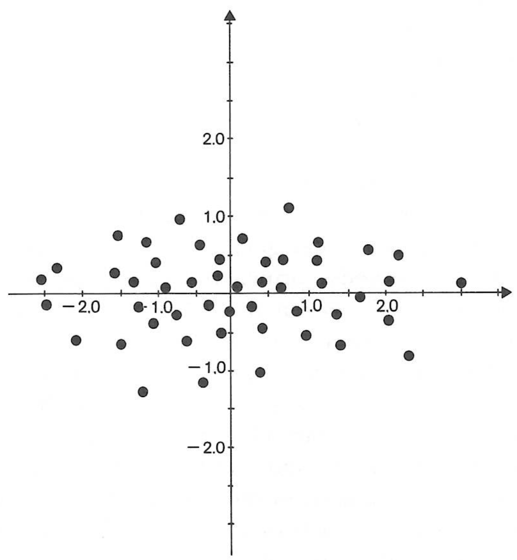

Figure 4. The same data as in Figure 3, but with the axes rotated in order to let one axis describe the largest possible part of the variation in the point scatter.

Contrary to normal regression, the orthogonal regression is mathematically complicated to carry out, but this need not concern us here. The line obtained will be the one of all possible lines on which the projections of the points have a maximal dispersion along the line. The line thus represents the maximum variation in a single dimension in the scatter, and gives so to speak a maximal 'explanation'.

The remaining variation in the two-dimensional example can be fully explained by a line perpendicular to the first through the origo. Indeed we may claim that we have merely rotated our two original axes of description in such a way that the first describes the maximal part of the variation in the scatter of points, while the second covers the rest (Figure 4). It is important to note that the interrelationship of the points has not been affected at all. Structure and distances in the point scatter have remained unaltered. It is only the original variables describing the points that have been replaced with others that maximize certain criteria of representation.

It causes no trouble to acknowledge that this two-dimensional example can be replaced by a three-dimensional one. Indeed we can place a line through the point scatter in a tri-axial coordinate system that meets the criteria of orthogonal regression. This line will represent the largest possible part of the variation in the point scatter that can be dealt with in one dimension. Further we can place a second line through the origo perpendicular to the first that represents the major part of the remaining variation, and finally we can place a third line through the origo perpendicular to the two others that covers the rest of the variation.

It is considerably harder to acknowledge that these principles also apply to a four-dimensional or indeed any multi-dimensional case. It makes no difference as far as the method is concerned whether we have three or a hundred dimensions. Arithmetically it works out fine. It is only our geometrical visualisation that is violated.

All three methods to be discussed here are based on orthogonal regression of point scatters in a multivariate metric space, and consequently they share some of the same characteristics in their ways of treating data. Notably, as orthogonal regression is based on minimization of the sum of squared distances from a set of points to a line, it tends to account for variance rather than correlation. This seems in general to be an advantage, but it does also mean that it is rather sensitive to uniqueness in the data, and consequently that careful data screening is a necessity in connection with its application.

The orthogonal regression itself is mathematically extremely complicated. It is performed by what is termed a spectral decomposition of a square (m x m) matrix (singular value decomposition of a rectangular (m x n where $m >= n$) matrix in the case of CA) with a content that properly represents the relationships between the points in the m-dimensional metric space as created from the data matrix.

The decomposition yields a number m non-negative numbers, the so-called eigenvalues or latent roots, and m corresponding vectors, the so-called eigenvectors or latent vectors. These vectors represent exactly the set of mm orthogonal regression lines to be found. The corresponding eigenvalues represent the amount of variation covered by the individual vectors. That is to say, the proportion that a single eigenvalue constitutes of the sum of all eigenvalues is equal to the proportion of the total amount of variance represented by the associated eigenvector. The vectors can be ordered by the eigenvalues in falling order, so that the first vector, called the first principal axis, explains the largest part of the variance compared to the other vectors, the second principal axis the largest but one part of the variance, and so forth.

The difference between the three methods of multivariate analysis is outlined in the following.

Principal components analysis

In PCA the square input matrix for the spectral decomposition is the covariance or the correlation matrix between the variables in the data matrix. This implies that PCA can be used safely only with data to which it is meaningful to apply the concepts of covariance and correlation, and this is true with reasonably normally distributed measurement data only. Indeed, if we plot any two variables involved in the analysis against each other in a two-dimensional scattergram, we should find that the scatter of points is more or less an ellipsoid, with only a few outliers. Tendencies for curved point scatters, or scatters divided along two or more lines of correlation are, not acceptable in data used as input for a PCA. Variables that do not follow the normal distribution in general should either be removed from the analysis, or measures should be taken to ensure normality. Likewise, units with extreme values in one or more variables should be removed, since extreme observations will tend to dominate the first axis.

The PCA method is scale-dependent, when applied to a covariance matrix. This means that the actual numerical size of the scatter of values in the variables influences the result. It is thus not immaterial whether we measure a variable, say length, in cm or mm. By measuring in cm with one cipher following the decimal point instead of in mm, we shrink the scatter of this variable by a factor of 10 and reduce the importance of the variable accordingly.

This PCA scale-dependence makes it extremely important that questions of compatibility between variables are always carefully considered before analysis. The numerical scatter within the individual variable should always be catered for according to the importance of the variable, and co-analyses of incompatible variables should never be carried out using a covariance matrix.

A way around the problem with differential scaling is to use the correlation matrix as input. Here all variables become standardised and consequently expressed on the same scale. This solution is probably preferable in many contexts, but as all variables are given equal importance, there is always the danger that small unimportant variables are upgraded in importance beyond reason. The use of PCA then calls for extreme care and consideration, when the data are prepared for analysis.

Apart from the eigenvalues and the eigenvectors, the output from PCA consists of two tables, normally named factor or component scores and factor or component loadings, respectively.

The component scores are the coordinates of the points (units) on the new set of principal axes defined by the eigenvectors. The component loadings are a set of correlation coefficients between the new principal axes and the original variables. The component scores thus give the position of the points in the multidimensional space seen from another viewpoint than was the case with the original variables. The component loadings tells us to what degree the new principal axes are related to or representative of the original variables.

PCA is normally termed an R-mode type of analysis. This means that basically it analyses the interrelationship between variables. To a certain extent, the component scores can be said to describe inter-object relationships, but they cannot be referred to directly in terms of inter-object similarity.

Principal coordinate analysis

The PCO can with some justification be thought of as a Q-mode PCA of a similarity matrix of some kind. Q-mode here means that it is the interrelationship of the units that is being analysed. This interrelationship is based on the concept of similarity. Many measures of similarity may be used. All those cited by Sokal and Sneath (1973) will do, and perhaps most profitably the coefficient proposed by Gower (1971), that allows for a mixture of variables on all scales in the data matrix. The similarity coefficient matrix is a square symmetric matrix with the units in both rows and columns, and a metric measure of similarity in the individual cells between all units. Properly normalised (by subtracting from each cell the mean values of the corresponding rows and columns and adding the grand mean of the matrix) to secure positive semi-definite properties, this matrix may be submitted directly to spectral decomposition. The result of this will be an orthogonal regression on a (fictive) set of metric variables expressed through the similarity coefficients. The original variables, which need not be metric, do not enter the analysis, and are lost.

The, say n, units analysed are thus thought of as n vectors describing a set of variables in an n-dimensional space, and the axes (vectors) in this space are given by the similarity coefficients (similar to the correlation coefficients in the PCA method). With orthogonal regression, we get a new set of n vectors that describe the same set of variables in a new n-dimensional space, where each new axis represents a linear combination of the original ones.

Next we may investigate how the old axes correlate with the new ones by projecting all vectors in the original space onto the vectors in the new one. In this way we find the position of all units in relation to the set of principal axes, and naturally we are interested only in the first few principal axes that hold the major part of the information.

As each unit in a Q-mode analysis represent a dimension, and as the units are normally much more numerous than the variables, there often arise computational problems. More than a couple of hundred units can seldom be analysed in one run on most computers.

The use of a similarity coefficient that through composite calculations establishes the units in a metric space of reference has one disadvantage. It is not possible in a simple way, as with the PCA, to 'reverse' the process once the analysis has been carried out, and investigate the variables in relation to the new axial representation. We are not able to see 'what caused what'.

Correspondence analysis

With PCA categorised as an R-mode technique and PCO as a Q-mode technique, the CA may best be classified as a simultaneous R-mode and Q-mode technique. Its origin lies with the study of two-dimensional tables of contingencies, and consequently its extension to cover multivariate cases is also restricted to categorical data. This, however, is the only a priori restriction. As input to CA, any type of categorisation will do. We may use counts, presence-absence registrations, or just registrations of a presence among a series of alternatives. In the latter two cases, presence is noted as 1 while absence and excluded alternatives are noted as 0. The area of application may even be enlarged to cover continuous data by way of proper categorisation of these (Hill 1974).

From this it may be acknowledged that CA is a potentially very useful method. Not only does it work simultaneously on an R-mode and a Q-mode basis, but it also deals with types of variables that are extremely common in archaeology. Furthermore, it does this without assumptions concerning the distribution of the variables. We need not have poisson distribution attached to the error structure as with log-linear analyses of contingency tables, nor do we for that matter need to know beforehand the structure of the phenomenon under study.

Algebraically, CA presents an extension to the PCA (and PCO). Spectral decomposition was there applied to symmetric matrices to yield one set of eigenvectors and one set of eigenvalues. In CA the spectral decomposition, here called the singular value decomposition, is applied to a rectangular non-symmetric matrix, in which case two sets of eigenvectors and one set of eigenvalues are obtained. The one set of eigenvectors will represent R-mode and the other Q-mode. Provided that the data matrix is properly scaled, i.e. it shows similarity between variables and between units simultaneously on a mutual scale, then the vectors given in the R-mode solution and the vectors given in the Q-mode solution will refer to the same principal component space.

Naturally, the two sets of vectors may be found individually, as an R-mode spectral decomposition of the 'correlation' matrix (data matrix pre-multiplied by its transpose) and as a Q-mode spectral decomposition of the 'similarity matrix' (datamatrix post-multiplied by its transpose). This, however, would give the same problem with size as in the PCO method, and as the scaling of the datamatrix prior to analysis makes the spectral decomposition yield vectors referring to the same component space, it is in fact possible to calculate the one set of vectors from the other. We thus need to find the eigenvectors of the minor of the two symmetric matrices only, whether it is the one obtained by pre- or by post-multiplication with the transpose.

The scaling procedure itself is rather complex, and it lies beyond the scope of this paper to discuss it in detail. The first step is to reduce the data matrix to unity by division of each cell frequency with the grand total of the matrix. The sum of all cells thus becomes 1, and we may look upon the matrix as representing a probability distribution of the data. If we call this matrix P, we now enter the row marginal sums and the column marginal sums of P into two diagonal matrices, and subsequently pre-multiply P with the square root of the inverse of the row sum matrix and post-multiply it with the square root of the inverse of the column sum matrix. The effect of this transformation is to stretch, differentially, the column vectors by the reciprocal of the square root of their column sum and stretch, differentially, the row vectors by the reciprocal of the square root of their row sum. Variables measured on disparate scales are thus differentially weighted and, as it were, equalised; similarly for the objects (Jöreskog et al. 1976). The matrix we obtain by this transformation will have the required qualities for singular value decomposition, where the two sets of vectors refer to the same component space.

As stated at the beginning of this chapter, the three methods outlined here constitute a set of methods that can handle all types of variables that we may come across in archaeology. This, however, is not the same as saying that we can freely apply them on any dataset we come across.

To use these methods on actual archaeological data is far from simple, as will be demonstrated in the following two sections, where two specific areas of application are investigated in detail.

In the first section we will look into the study of artefact form based on measurements, and in the second at the classical seriation problem.

POTS FOR GOOD MEASURE

There has always been something fascinating about measurements, and archaeologists really love them. The joy of being able to state the unrefutable fact that a flint axe is 21.6 cm long, give or take half a millimetre, is surpassed only by the joy of being able to state that the exact mean length of say 200 axes is 19.6781 cm. The exactness of the statements, however, does not add to the amount of information gained on prehistoric societies. We receive exact information concerning size, but how does that lead to useful cultural information? In the case of flint axes it may tell us only how worn down an axe became before it was discarded, and only combined with other information can this be of any help.

Measurement data become much more useful when they are used to describe form rather than size. Basically, a description of form means that we take two or more measurements on each item and make comparisons between the items based on these sets of measurements. This of course seems to point straight into multivariate analysis, and from what was outlined in the foregoing chapter it would be natural to apply either a PCA or a PCO to such data.

However, life is not that simple, nor is the treatment of measurement data. If we merely take a series of basic measurements that gives a general outline of the items to be analysed, and take these measurements as direct input for a PCA or PCO, then we are in for great trouble as far as information on form is concerned. Whallon (1982) has demonstrated this very effectively, and has reached very depressing conclusions concerning the usefulness of multivariate statistics in the study of form based on measurements.

He used a series of jars, pitchers, and necked bowls from the Swiss Late Neolithic site of Niederwil to exemplify the problems that arise when continuous measurement data are used as input for a PCA with the hope of finding the base for a workable formal typology (1982:140). As input data, he chose 10 measurements of various diameters and height on the pots, as well as one measurement representing volume. A correlation matrix (1982:Fig 6.2) based on these measurements shows a high all-over positive correlation between the variables. There is thus a high redundancy among them, and obviously this stems from the size of the pots, the measure of volume having by far the highest average correlation with the other variables.

Clearly, size is bound to dominate most analyses carried out with these 11 variables in their raw state, as is the case when they are subjected to a PCA (1982:Fig. 6.3). The first axis accounts for over 83% of the total variance, and it has extremely high loadings on all variables. Indeed, it is the only component with an eigenvalue higher than one, and following normal procedures, the resultant claim should be that the material has only one underlying dimension determining its form, namely that of size. Looking at the outline drawings of the pots, this is quite obviously nonsense, and if one takes the second principal component into consideration, it turns out that it does indeed hold information concerning form, despite its low eigenvalue. A closer inspection of the 1st and 2nd principal components together shows, however, that a proper separation according to form only occurs among the small pots (1982:Fig. 6.11). The reason for this failure can be found by plotting the variables against each other two by two (1982:Fig. 6.12). It then turns out that some combinations of variables do not centre around a simple ratio (regression line) as they supposedly should do. The data at hand are in fact unsuitable for a PCA as they are, and by simply plotting the direct measure of pot size against the ratio between neck height and neck diameter, Whallon can create a division that is much more accurate than the one he obtains by PCA (1982:Fig 6.15).

Whallon states in his general discussion that these problems with redundancy due to size and two or multimodal relationships among primary measurement variables is the rule rather than the exception in studies of artifact form. This makes the use of PCA hazardous if it is not applied with the utmost care. Screening and transformation of the data may indeed be necessary. In general, Whallon seems to prefer not to use multivariate statistics in connection with studies of form based on measurement data. Instead he is inclined to rely on more simple two-dimensional methods.

This depressing example should not prevent measurements on a number of pots being analysed here using a PCA. However, the lessons learned from Whallon will be borne in mind, and hopefully it will be possible to apply the PCA successfully if the proper precautions are taken.

The material to be analysed comes from a study by Eva Koch Nielsen (1983). It consists of 135 complete pots from the Early Neolithic and the earlier Middle Neolithic TRB Culture on the islands east of the Great Belt in Denmark. The pot profiles were placed in a two-dimensional coordinate system with the pots 'standing' on the horizontal axis, and with their vertical symmetry line coinciding with the vertical axis. Eight well-defined points along the profile of the pot were then measured giving rise to 16 separate measurements (Figure 5).

The pots measured comprise three main pottery forms: funnel beakers, bowls and flasks. This was acknowledged from the outset using the standard definitions of these forms, and it turned up very clearly in the two-dimensional scattergrams of the basic measurements also. An example of this can

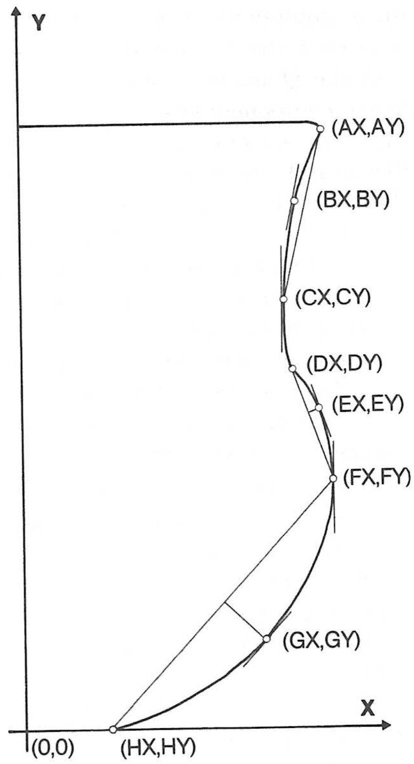

Figure 5. Measurement points of pot profiles used by E.K. Nielsen 1983.

be seen in Figure 6, where the diameter of the rim is plotted against the height of the pot for funnel beakers and flasks. It is obvious that we are dealing with quite different forms, and rather than trying to analyse them together, Nielsen wisely decided to treat them separately, thereby avoiding one of the pitfalls discussed by Whallon.

The largest group of pots in the material is the funnel beakers comprising a total of 102 pots. These alone will concern us in the following. Nielsen continued her investigations by comparing the pot profiles. In order to make these immediately comparable, her first step was to scale all measurements with pot height as unity. She proceeded by calculating a coefficient of agreement between each profile using a squared distance measure. Finally, she created a minimum spanning tree from the matrix of agreement coefficients, using a method very much like the one proposed by Renfrew and Sterud (1969).

The minimum spanning tree formed the basis for her division of the funnel beakers into formal groups, although she did make a visual comparison of the profiles also, and as a result of this moved pots from one group to the other. She ended up with a total of 21 groups which could be assembled into six main groups. Finally, she divided the material into a series of types, which as their base had her formal groups, but which also included the presence or absence of lugs, and various forms of decoration. Thus her final typology was not entirely morphological.

Her typology, when compared to available C-14 dates and information on find contexts, has given much new and valuable information to problems of chronology and group divisions in the Early Neolithic. That, however, is not the issue here. What is of interest in the current context is that by the method she used she did get to the very point of analysing detailed formal variation, and did obtain a result that made sense archaeologically. It would therefore seem worthwhile to try out a PCA on the same data to see how far we may go using that method, and whether it can confirm Nielsen's results.

In order to be able to compare the profiles the way she did, Nielsen had to scale the measurements. Thereby she removed the factor of size from the data. This, in my opinion is the main reason why her analysis was so successful. In order to get a good result from a PCA, it is likewise imperative that the influence of size be removed. In fact, a PCA of the raw measurements (Table 1) yields an explanation percentage as high as 95% for the first component, and if we plot the scores against the third root of the volume (volume is a measure of third degree), we get a correlation coefficient of 0.99. Volume is thus the dominating factor in the material, and its presence prohibits any concern with morphological variation.

One obvious way to scale the measurements would be with the aid of the third root of the volume. This

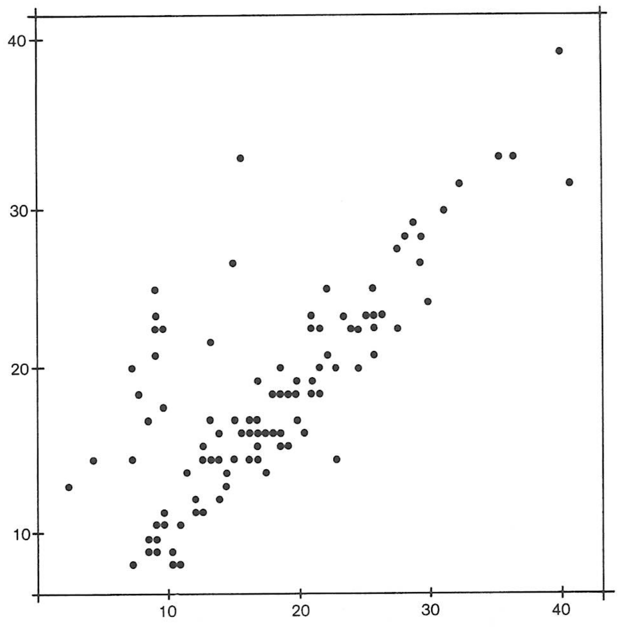

Figure 6. Diameter of rim plotted against height of pot for funnel beakers and flasks.

I have tried with good results, but have nevertheless chosen to use the same method as Nielsen with a minor modification. She scaled all measurements to the height of the pots, but admits that she had difficulties in her comparisons with very narrow and very wide pots. To avoid this, I have scaled all vertical measurements to the height of the pots and all horizontal measurements to the width of the rim. In terms of comparisons of profiles this means that all profiles start and end in the same two points. It may not reduce the effect of size completely, but it turns out to be very effective.

In Figure 7 the scores of the first two components are plotted against each other with signatures according to Nielsen's type divisions. Together they cover 73% of the total variation, and they both have eigenvalues well above one (Table 2). If we take the individual components, then the first component (horizontal axis in the plot) has positive loadings exclusively. Remembering the way the measurements were scaled, this means that positive scores on the first axis indicate that all measurement points are high compared to the total height, and wide compared to the rim diameter. Negative scores on the other hand mean that all measurement points lie low compared to the total height and close to the symmetry line compared to the rim diameter. The second component shows an interesting pattern, since the loadings of all vertical measurements are positive, while the loadings of all horizontal measurements are negative. Consequently, pots with positive scores tend to be flaring with high vertical measurements (particularly the belly measurements), whereas the pots with negative scores have broad horizontal measurements (especially those of the belly) resulting in round-bodied pots.

A closer look at the pattern in Figure 7 in relation to Nielsen's divisions suggests that the result of the PCA is not optimal. It is quite obvious that problems are attached to the 'MN types', which are exclusively distributed along the first component. Probably, the presence of pots of this type possessing high negative scores completely determines this component. As the points of main interest in Nielsen's study lie with the Early Neolithic pottery, and as the Middle Neolithic pottery can be separated from the rest using decorational rather than morphological criteria (which Nielsen used to separate them as 'MN types'), the most logical step is to remove them from the study. Consequently, a new PCA was performed that did not include the MN pots plus a few other pots considered by Nielsen (personal communication) to be very atypical.

A total of 81 pots were analysed this time, resulting in three components with eigenvalues higher than one (Table 3). The scores of the two first components plotted against each other are seen in Figure 8 cover-

J	AX	AY	BX	BY	CX	CY	DX	DY	EX	EY	FX	FY	GX	GY	HX	HY
A																
15	4.32	8.65	3.92	7.25	3.72	5.75	3.82	4.95	4.22	4.07	4.32	3.22	3.10	0.92	0.00	0.00
23	11.40	20.00	10.37	17.42	9.60	15.20	9.60	15.20	10.20	13.95	10.37	11.87	8.77	5.35	4.37	0.10
32	8.15	17.00	7.35	15.70	6.65	12.82	7.02	11.42	7.92	9.67	8.20	7.87	5.80	1.95	0.00	0.00
37	10.60	19.20	9.50	17.05	9.03	14.07	9.33	12.33	10.27	10.97	10.43	9.37	7.83	2.67	0.00	0.00
38	7.53	14.20	6.67	11.40	6.40	8.72	6.45	7.90	7.00	6.90	7.17	5.92	5.97	2.75	2.32	0.05
43	6.80	12.10	6.27	10.33	5.97	7.83	6.10	7.30	6.40	6.20	6.55	5.10	5.70	2.00	3.50	0.30
45	8.05	15.60	7.52	12.67	7.20	10.20	7.35	9.52	8.30	7.97	8.60	6.65	7.17	2.55	3.97	0.22
46	8.80	13.77	8.22	12.45	7.70	11.15	7.70	11.15	7.80	10.87	7.90	10.55	6.25	4.37	2.47	0.35
52	7.53	17.20	6.67	15.13	6.00	10.02	6.42	9.02	7.10	8.05	7.40	6.55	6.10	2.95	2.37	0.10
65	8.40	14.82	7.97	13.10	6.85	10.35	6.85	10.35	7.52	8.70	7.75	6.97	6.45	2.80	4.07	0.20
82	9.30	18.70	8.37	16.20	8.00	13.27	8.62	10.27	9.42	9.37	9.70	8.20	7.65	3.75	2.10	0.10
86	6.97	15.90	6.40	14.17	5.87	11.93	5.93	11.77	6.83	10.00	7.30	6.73	6.17	2.70	3.60	0.20
87	6.35	14.20	5.95	11.65	5.63	9.17	5.77	8.77	6.37	7.77	6.65	6.07	5.45	2.52	2.75	0.07
88	10.43	18.20	9.27	15.50	8.93	13.93	9.23	12.47	9.67	11.50	9.90	9.27	8.50	4.05	4.43	0.07
91	8.25	16.50	7.70	14.55	7.25	12.05	7.85	8.95	8.65	7.70	8.95	6.15	7.40	2.20	3.75	0.05
93	7.90	16.20	7.10	14.00	6.80	11.33	7.22	9.27	7.85	8.15	8.12	6.27	6.40	2.40	2.40	0.00
94	8.50	19.07	8.00	16.93	7.73	14.63	8.10	10.87	8.90	9.60	9.27	7.77	7.53	3.10	2.82	0.30
99	4.50	10.20	4.07	7.95	3.90	6.57	4.00	6.10	4.45	5.20	4.62	4.17	3.92	1.77	2.10	0.00
181	7.82	16.30	7.05	14.35	6.75	12.80	6.85	12.07	7.37	10.35	7.57	8.27	6.22	3.37	3.02	0.20
182	5.70	13.20	5.12	10.65	5.00	9.02	5.32	7.50	5.82	6.75	6.00	5.70	4.75	2.10	2.07	0.20
183	8.47	18.90	7.63	15.77	7.40	14.10	7.47	13.43	8.63	11.73	9.12	8.82	7.45	3.82	3.57	0.25
184	6.77	14.20	5.90	11.77	5.50	10.27	5.62	9.77	6.40	8.20	6.62	6.47	5.37	2.52	2.80	0.03
185	5.10	8.10	4.32	5.92	4.10	5.15	4.10	5.15	4.10	5.15	4.10	5.15	2.90	1.47	0.00	0.00
186	6.60	16.50	6.10	14.00	5.90	11.60	6.07	11.40	6.97	10.00	7.40	7.50	6.02	2.77	2.95	0.00
187	9.23	20.00	8.53	18.03	8.03	15.33	8.03	15.33	9.27	13.40	9.60	10.87	7.95	5.02	3.80	0.12
188	8.20	14.10	7.67	12.10	7.22	9.92	7.47	9.42	7.80	8.65	7.92	7.32	6.63	3.00	3.00	0.02
189	3.70	8.30	3.30	6.95	3.10	5.40	3.25	4.20	3.65	3.65	3.85	2.90	2.95	0.90	1.20	0.00
190	5.33	10.20	5.07	8.77	4.87	7.40	5.00	4.90	5.30	4.33	5.43	3.70	4.10	1.27	1.30	0.03
196	8.92	16.10	7.60	13.10	6.80	9.95	7.05	9.22	7.72	7.97	7.95	6.62	5.40	1.75	0.00	0.00
200	5.43	8.20	4.50	6.80	4.20	5.83	4.17	5.60	4.30	5.15	4.32	4.57	3.67	1.97	1.55	0.30
201	6.30	14.00	6.00	12.20	5.17	9.80	5.17	9.80	6.20	8.63	6.83	6.43	5.60	2.80	2.57	0.12
202	5.00	7.60	4.40	6.50	4.00	5.70	4.00	5.70	4.10	5.60	4.10	5.40	3.30	2.30	1.70	0.00
205	14.10	28.30	12.70	23.40	12.40	20.70	12.53	19.70	13.67	17.73	14.05	14.82	11.10	5.77	5.32	0.40
207	4.70	10.00	4.15	8.00	4.00	6.55	4.10	4.80	4.45	4.40	4.55	3.60	3.85	1.45	1.80	0.10
209	12.42	20.40	11.12	16.95	10.60	14.52	10.87	13.17	11.87	11.80	12.27	9.67	8.52	2.52	0.00	0.00
210	4.60	9.70	3.90	8.00	3.62	6.45	3.72	5.67	4.07	4.97	4.20	4.12	3.55	1.85	1.70	0.02
214	6.25	11.00	5.72	9.75	5.47	8.97	5.55	8.77	5.57	8.40	5.60	8.15	4.32	3.17	1.52	0.15
221	12.67	23.10	11.57	20.10	11.02	16.75	11.70	13.92	12.97	12.02	13.23	10.37	10.72	3.90	5.37	0.05
222	13.67	27.00	12.10	22.67	11.57	19.20	11.95	16.87	12.87	14.60	13.13	12.27	10.40	4.90	4.60	0.00
223	12.83	22.00	11.87	18.87	11.57	16.40	11.77	14.97	12.63	13.60	12.87	11.87	10.03	4.73	4.80	0.13
225	4.97	9.00	4.35	7.70	4.10	6.72	4.10	6.70	4.27	6.42	4.35	5.82	3.65	2.57	2.02	0.02
227	7.33	13.90	6.47	11.53	6.10	8.53	6.17	7.93	6.63	7.23	6.85	5.92	5.70	2.65	2.52	0.05
229	4.62	9.60	4.12	8.17	3.92	6.67	3.97	6.00	4.45	5.07	4.65	4.02	3.85	1.70	1.77	0.02
230	10.50	23.50	9.15	19.22	8.50	16.55	8.50	16.55	10.07	14.47	10.65	11.47	8.72	4.85	3.82	0.05
249	8.37	16.40	7.37	13.17	6.92	10.15	7.00	9.57	7.52	8.52	7.72	7.22	6.50	3.25	3.12	0.13
250	18.20	32.50	16.35	30.40	14.75	26.10	14.90	25.30	15.40	23.70	15.55	22.10	12.10	8.00	5.10	0.40
251	13.05	25.20	11.85	21.75	10.70	19.25	11.15	18.75	11.40	17.85	11.45	16.30	8.75	6.40	6.00	0.50
255	17.55	32.50	15.30	29.20	14.60	25.95	14.80	25.35	15.55	23.60	16.00	21.10	12.55	7.60	5.60	0.05
259	9.10	18.40	8.20	15.35	7.85	13.95	8.20	13.55	8.75	12.20	9.00	10.20	6.95	4.15	3.40	0.00
260	6.10	11.30	5.70	9.90	5.20	8.50	5.25	8.30	5.60	7.55	5.60	6.90	4.60	3.05	1.65	0.00
262	8.60	13.80	7.55	11.90	7.20	10.70	7.20	10.40	7.35	9.90	7.40	9.20	5.75	4.15	2.30	0.20
267	6.43	14.50	6.00	11.97	5.80	10.47	6.10	8.97	6.60	7.73	6.75	6.62	5.35	2.57	2.52	0.20
268	9.87	18.10	9.30	16.30	9.07	14.80	9.17	14.07	9.33	13.37	9.43	12.20	7.97	5.23	4.03	0.17
271	14.83	26.60	13.17	23.87	12.57	22.33	12.57	22.20	13.53	19.87	13.73	18.33	11.10	7.50	5.17	0.07
272	4.30	9.40	3.70	7.60	3.40	5.70	3.80	5.00	4.20	4.50	4.40	3.60	3.60	1.80	1.85	0.00
273	11.00	20.60	10.40	18.30	9.60	15.40	9.80	14.20	10.50	13.00	10.90	10.00	8.40	3.60	4.30	0.00
274	5.90	11.70	5.60	10.10	5.07	8.63	5.17	8.40	5.47	7.17	5.63	5.80	4.67	2.27	2.12	0.20
278	15.50	29.50	13.90	24.25	13.40	20.20	13.90	19.10	15.60	16.45	16.25	13.25	12.70	4.80	6.00	0.20
279	10.60	22.40	9.37	18.85	8.92	16.50	9.22	16.10	9.80	14.43	10.20	11.45	8.25	4.55	3.85	0.25
281	4.50	8.60	3.90	7.20	3.65	5.82	3.70	5.12	4.20	4.47	4.45	3.40	3.75	1.45	2.00	0.10
283	10.80	22.40	9.73	19.87	9.07	17.93	9.37	16.90	10.10	15.57	10.32	13.22	8.82	5.85	4.52	0.22
288	13.85	22.60	12.10	19.10	11.30	17.07	11.40	16.13	11.90	13.70	12.03	11.60	10.50	5.80	4.07	0.30
290	12.80	22.90	11.80	19.65	11.47	17.72	11.92	15.37	12.95	13.30	13.25	11.37	10.57	4.67	4.90	0.25
293	12.90	22.00	11.20	17.70	10.70	14.90	11.00	13.30	12.10	11.20	12.40	7.50	10.90	3.40	5.10	0.00
294	10.70	19.90	9.20	16.30	8.80	14.10	8.90	13.75	9.65	12.20	9.85	10.40	7.70	3.55	3.75	0.05
300	10.20	16.10	9.70	13.10	9.37	11.57	9.45	10.02	9.62	9.70	9.75	9.05	6.87	3.70	3.12	0.07
305	9.20	15.10	7.57	11.50	6.72	8.17	6.75	8.10	7.35	7.07	7.57	5.87	6.75	3.30	3.60	0.00
307	12.77	21.10	11.17	17.13	10.27	13.10	10.53	11.10	11.03	10.00	11.23	8.80	9.20	3.93	4.02	0.12
308	8.35	14.10	7.50	10.90	6.95	8.60	7.10	7.80	7.45	7.05	7.45	6.05	6.35	3.10	3.45	0.20
311	9.53	15.00	8.47	12.05	8.07	8.87	8.10	8.30	8.22	7.80	8.25	7.17	6.50	2.67	2.62	0.05
312	6.95	15.90	6.45	12.87	6.17	10.87	6.27	9.92	7.25	8.20	7.42	6.90	6.30	3.17	3.17	0.15
320	14.77	28.00	13.47	23.20	12.92	19.55	13.15	16.67	14.20	14.87	14.57	12.52	12.22	5.97	5.00	0.27
321	4.40	10.20	3.92	8.10	3.75	6.07	3.77	5.57	4.20	5.02	4.45	3.72	3.07	1.05	0.00	0.00
323	16.20	31.20	15.40	26.53	15.03	23.03	15.13	22.27	16.17	19.77	16.53	16.23	13.20	6.60	3.87	0.00
324	6.37	11.60	5.83	9.80	5.57	8.65	5.72	8.15	6.10	7.20	6.30	6.10	5.10	2.12	2.65	0.17
325	20.30	31.50	18.00	27.35	16.15	23.90	16.20	23.45	16.75	21.35	16.97	18.60	13.72	8.05	5.95	0.27
326	4.80	11.20	4.53	9.33	4.40	7.55	4.70	6.52	4.97	6.00	5.13	4.73	4.23	2.03	2.22	0.05
327	11.05	24.80	10.17	21.67	9.92	19.97	10.42	18.60	11.22	16.75	11.57	14.62	9.80	7.22	4.20	0.32
328	10.47	23.00	9.77	19.83	9.27	17.30	9.50	16.62	10.40	14.70	10.85	11.45	9.02	5.32	3.87	0.07
329	12.35	22.20	12.10	19.97	11.85	16.62	12.05	15.17	12.50	14.70	12.67	14.10	9.10	5.35	3.25	0.00
336	7.00	12.10	5.70	9.45	5.32	7.95	5.37	7.80	5.42	6.95	5.45	5.95	4.50	2.60	2.50	0.27
B																
3	9.63	18.73	6.87	14.40	5.52	8.97	5.52	8.97	6.52	8.10	6.90	6.90	5.62	3.72	2.10	0.15
33	14.40	29.20	13.75	26.35	13.50	23.20	13.95	20.20	14.55	19.25	14.75	17.55	12.50	9.05	5.85	0.00
59	11.92	22.10	10.45	17.30	9.90	14.22	9.90	14.22	10.32	12.97	10.45	10.70	8.00	4.57	3.20	0.05
60	8.40	14.40	7.85	11.85	7.60	9.65	7.65	9.15	7.85	8.65	7.90	8.00	6.35	2.85	3.50	0.25
61	13.20	23.10	12.50	19.90	12.10	16.90	12.10	16.90	12.70	15.40	12.80	13.60	11.40	9.20	4.20	0.00
131	11.60	23.00	10.77	19.77	9.90	16.67	10.25	15.27	11.22	13.87	11.70	11.82	9.72	5.22	4.97	0.30
195	9.40	16.30	8.32	13.25	8.05	11.47	8.17	10.37	8.57	9.37	8.70	8.00	7.17	3.52	2.97	0.00
211	7.27	12.80	5.62	10.30	4.52	7.02	4.55	6.60	5.12	5.97	5.35	5.17	4.30	2.30	1.67	0.05
212	8.27	14.70	7.97	13.13	7.60	10.47	7.65	9.27	7.77	8.97	7.82	8.60	6.52	3.92	3.10	0.03
213	6.50	14.00	5.00	11.00	4.00	6.83	4.00	6.70	4.77	6.07	5.07	4.97	4.10	2.10	2.17	0.00
216	7.00	14.70	5.95	12.10	4.57	7.15	4.57	7.15	5.17	6.15	5.42	5.17	4.67	3.07	1.45	0.05
218	8.70	16.00	8.43	13.70	8.30	12.43	8.37	11.90	8.60	11.27	8.73	10.63	6.60	5.13	2.75	0.05
226	6.22	15.30	5.07	9.60	4.60	6.25	4.62	6.17	5.10	5.12	5.30	4.22	4.42	2.12	2.37	0.02
233	9.77	18.90	9.15	15.70	9.00	13.42	9.15	11.85	9.30	11.55	9.37	11.15	7.02	3.72	3.85	0.27
234	10.80	18.80	9.70	15.90	9.03	10.93	9.10	10.87	9.43	10.27	9.60	9.00	7.27	3.53	3.23	0.03
236	9.90	17.20	8.40	13.55	7.70	10.33	7.83	9.97	8.10	9.03	8.20	8.22	6.92	3.80	3.52	0.20
286	12.02	22.50	11.42	19.65	11.27	17.20	11.37	14.97	11.57	14.60	11.62	14.20	9.47	6.80	3.50	0.15
292	15.10	24.20	13.17	19.97	12.27	15.87	12.43	14.97	12.77	14.03	12.92	12.45	10.32	5.72	4.55	0.20
306	20.23	39.40	18.17	34.20	17.53	27.93	18.17	24.03	19.10	22.63	19.50	21.23	14.63	9.47	4.33	0.07
309	11.43	14.40	10.17	11.27	9.77	10.00	9.80	9.83	9.90	9.47	9.97	9.07	8.93	5.17	3.60	0.07
322	11.00	21.20	8.00	16.90	6.20	10.57	6.20	10.57	7.15	9.20	7.45	7.87	6.42	4.72	2.77	0.07
C																
17	7.90	32.80	7.35	27.80	7.02	23.17	7.37	22.75	11.57	19.32	13.47	12.25	10.92	4.15	4.82	0.17
26	4.80	17.65	4.50	15.47	4.25	11.70	4.97	10.72	6.67	9.12	7.22	6.87	6.07	3.07	2.77	0.18
28	3.70	14.30	3.23	11.77	3.20	8.90	3.20	8.90	5.33	6.83	5.93	4.83	5.07	2.30	2.53	0.20
42	2.13	14.60	2.07	14.00	2.32	8.87	3.02	7.50	4.50	6.85	5.20	5.35	3.55	1.37	0.00	0.00
56	7.60	26.70	7.30	23.00	7.00	20.10	7.10	16.90	9.80	13.70	10.60	10.00	8.70	4.30	4.05	0.20
110	4.10	16.50	4.00	14.40	4.20	11.30	5.10	10.42	6.52	9.10	7.05	6.85	5.62	2.50	2.80	0.10
113	4.00	18.50	3.40	18.50	3.70	16.55	3.80	14.75	6.35	11.85	7.00	7.35	5.60	2.30	3.70	0.15
117	6.70	21.50	6.05	19.45	5.70	14.85	5.70	14.85	8.65	12.25	9.30	9.30	7.05	3.20	2.15	0.00
219	4.95	17.60	4.35	14.82	4.12	13.05	4.65	11.87	6.55	9.77	7.27	7.07	6.10	3.12	2.82	0.07
237	1.27	12.90	1.17	11.20	1.57	9.07	2.55	8.22	5.05	6.65	6.10	4.97	4.25	1.42	0.72	0.00
238	4.45	21.10	4.15	18.97	3.95	15.02	5.00	11.30	7.92	8.97	8.72	6.80	7.10	3.37	3.37	0.12
239	3.60	19.90	3.20	16.50	3.10	13.63	4.53	10.60	6.40	8.40	6.93	5.93	5.83	2.20	3.13	0.00
240	4.47	24.50	3.90	21.77	3.67	18.50	4.25	13.55	8.00	11.05	9.40	7.82	6.95	2.47	1.22	0.00
241	4.42	23.40	3.70	19.45	3.42	13.50	4.90	9.95	7.75	8.65	8.52	7.07	6.92	2.87	2.95	0.03
242	4.55	22.00	4.32	19.75	4.10	16.20	4.60	12.40	7.60	10.30	8.72	6.85	7.17	2.57	3.40	0.10
245	4.95	22.00	4.65	20.10	4.37	17.85	5.60	14.00	8.17	11.40	9.17	8.63	7.37	3.73	3.20	0.03

Table 1. 16 measurements as defined in Figure 5 taken on 135 pots. The three groups of pots are: A: funnel beakers. B: bowls. C: flasks. Data after E.K. Nielsen 1983.

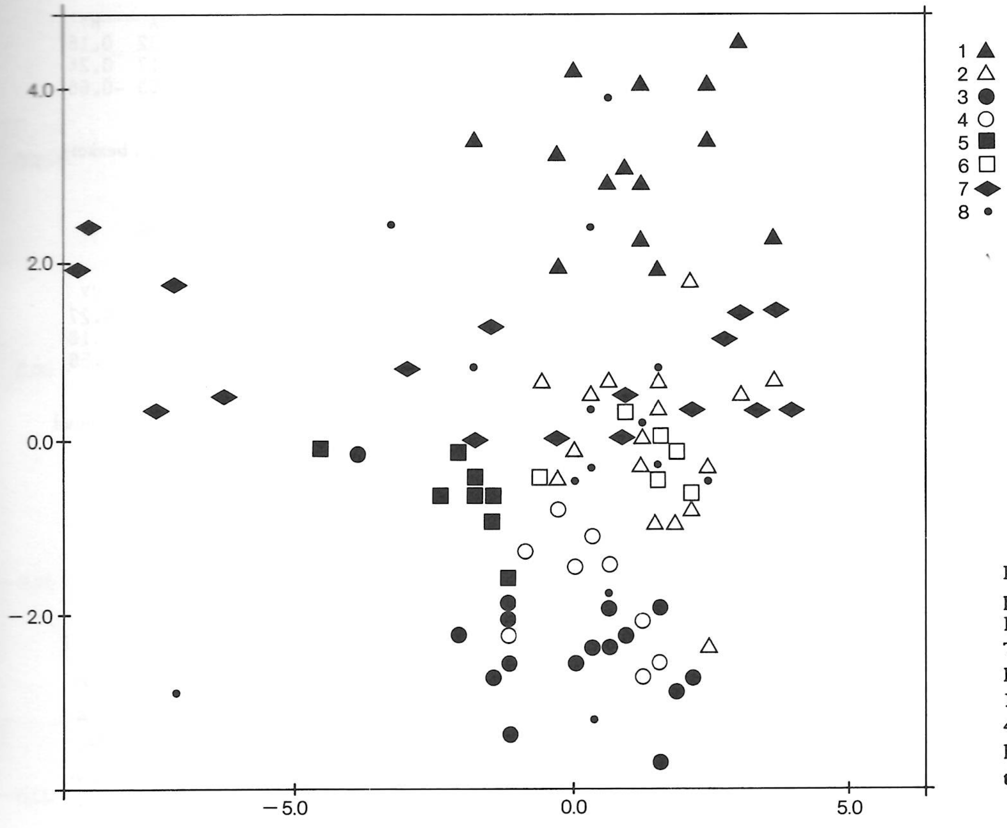

Figure 7. Plot of the first two principal components from a PCA of 102 funnel beakers. The numbering corresponds to Nielsen's type division as follows: 1: type I. 2: type II. 3: type III. 4: type IV. 5: type V. 6: broad-lugged beakers. 7: MN types. 8: type not decided.

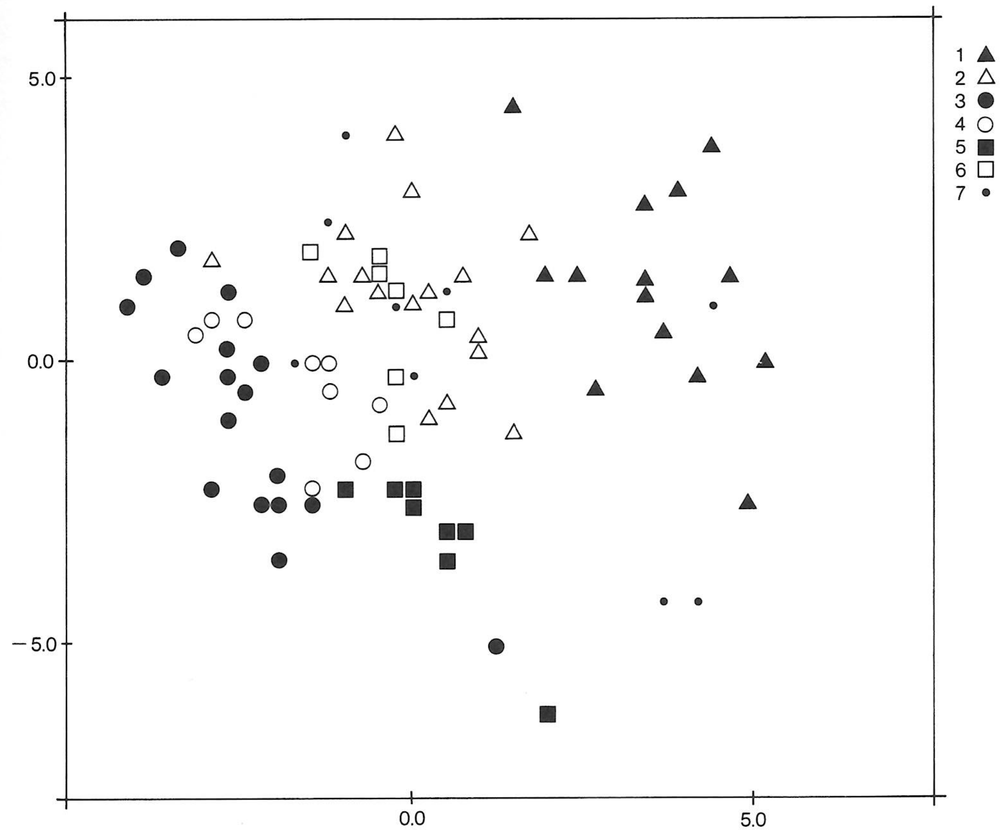

Figure 8. Plot of the first two principal components from a PCA of 81 funnel beakers. The numbering corresponds to Nielsen's type division as follows: 1: type I. 2: type II. 3: type III. 4: type IV. 5: type V. 6: broad-lugged beakers. 7: type not decided.

Axis	E.val.	Expl.%	BX	BY	CX	CY	DX	DY	EX	EY	FX	FY	GX	FY	HX	HY
1	6.62	47.28	0.89	0.69	0.89	0.88	0.87	0.70	0.78	0.67	0.71	0.55	0.63	0.37	0.32	0.15
2	3.65	26.05	-0.22	0.24	-0.24	0.32	-0.36	0.61	-0.60	0.71	-0.66	0.79	-0.59	0.72	-0.17	0.24
3	1.38	9.84	0.10	0.18	0.12	0.09	0.12	-0.03	0.08	0.03	0.06	0.08	-0.34	-0.20	-0.83	-0.66

Table 2. Eigenvalues, explanation percentages and factor loadings for the first three principal axes from a PCA on all 102 funnel beakers.

Axis	E.val.	Expl.%	BX	BY	CX	CY	DX	QY	EX	EY	FX	FY	GX	FY	HX	HY
1	5.22	37.30	-0.44	0.16	-0.53	0.36	-0.67	0.64	-0.86	0.72	-0.87	0.78	-0.73	0.69	-0.18	0.27
2	4.66	33.27	0.74	0.72	0.71	0.84	0.61	0.68	0.45	0.65	0.39	0.55	0.35	0.50	0.29	0.18
3	1.52	10.86	0.14	0.19	0.17	0.11	0.17	-0.03	0.06	0.04	0.01	0.12	-0.47	-0.27	-0.86	-0.58

Table 3. Eigenvalues, explanation percentages and factor loadings for the first three principal axes from a PCA on 81 selected funnel beakers.

ing 71% of the total variation. The two components are in fact the same as before, but they have changed places so that the first and most important component is the one that was second before and vice versa. Moreover, the plot of the scores now shows a clear tendency to cluster, that in part answers well with the division suggested by Nielsen.

The attempt to make a morphological division using basic measurements is thus quite successful, and it is not due to profound morphological differences in the material analysed, as can be seen from Figure 9, where examples of Nielsen's types I, II and III are shown. Especially, the marked separation of type I from II may come as a surprise to many who have been working with the Early Neolithic, and it is of utmost importance for the current discussion concerning chronology and group divisions at the beginning of the Early Neolithic.

Instead of the R-mode PCA that takes its starting point in the relationships between the measurements, we could also have used the Q-mode PCO that takes its starting point in the relationships between the pot profiles. In Figure 10 we see the coordinates of the two first components from a PCO plotted against each other. The input similarity matrix is based on squared distances between the pot profiles, computed in exactly the same way as Nielsen did. A comparison between the results of this PCO (Figure 10) and the foregoing PCA (Figure 8) shows them to be almost identical. Thus, even though it may seem more correct to rely on comparison of profiles (PCO), it turns out that a PCA based on interrelationships between the variables works out just as well. As the latter can handle far more cases in one run on the computer than the former (limited by the number of variables only), and as it is easier to interpret the components in a PCA by help of the loadings, then this method is far to prefer for analyses of this type.

ARCHAEOLOGICAL TIME SERIATION

Time is undoubtedly the most dominating issue in archaeology. It cannot be observed, yet it plays a role in the evaluation of any observation made. Change in past material culture happened on many different levels, and was caused by many different types of agents. It is one of the main goals of archaeology to outline and explain these changes in terms of dynamics and structure in specific prehistoric societies. Yet, in all societies that we are dealing with, we find that part of the changes follows some general pattern for which a specific historic explanation would miss the point.

This phenomenon was first discussed in depth by Hildebrand and Montelius in the second half of the preceding century (Gräslund 1974:167ff). By empirical observation they noticed that artifacts from one period show almost always close affinity in form to those of the immediately preceding period.

In line with the natural sciences of that time, they spoke of an evolution, and their theory was that this

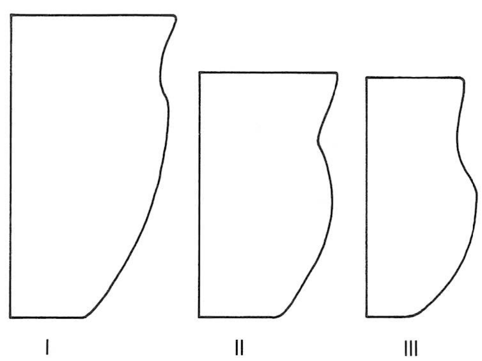

Figure 9. Examples of the three main type groups separated through the analysis Figure 8.

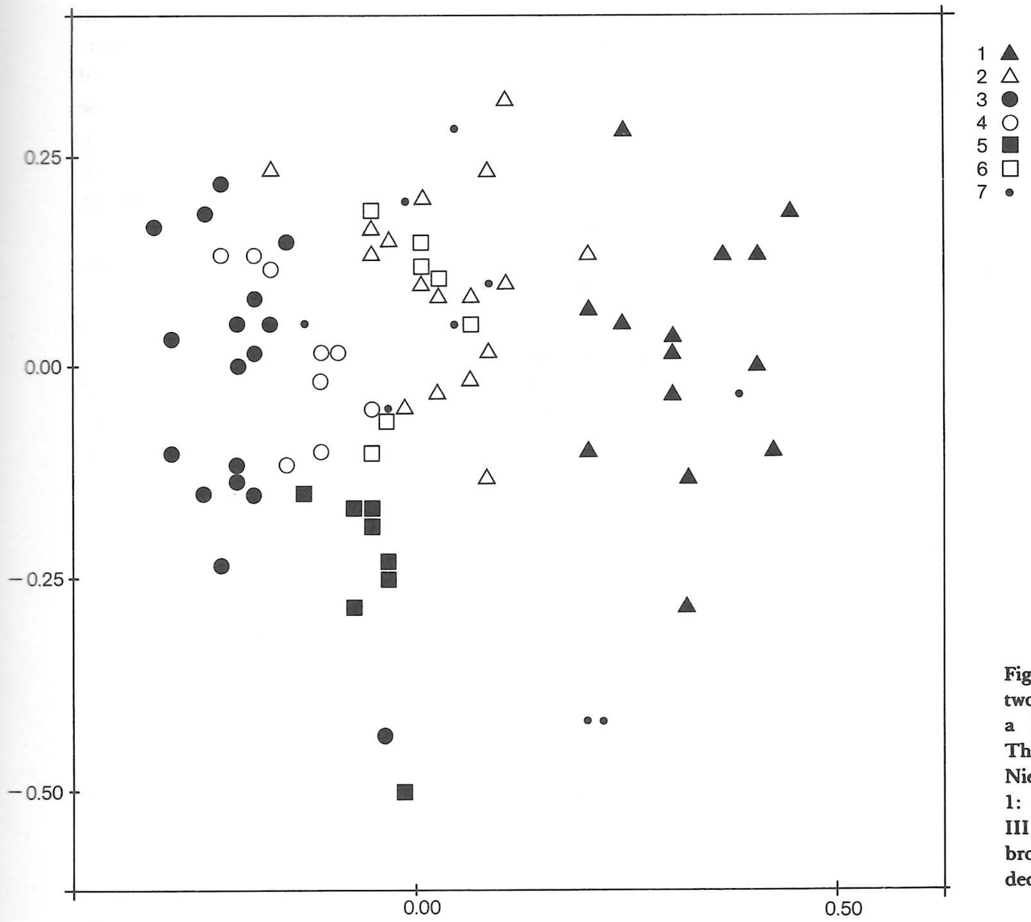

Figure 10. Plot of the first two principal components from a PCO of 81 funnel beakers The numbering corresponds to Nielsen's type division as follows: 1: type I. 2: type II. 3: type III. 4: type IV. 5: type V. 6: broad-lugged beakers. 7: type not decided.

evolution was continuous, unbroken and inherent to the cultural process. One practical outcome of this - and indeed their main concern - was that conclusions on time relationships could be obtained directly from a study of the artifacts without any reference to the find context (Gräslund 1973:19-20).

This was the beginning of the typology concept. Since then this concept has been treated to death by archaeologists, and nothing or very little remains of its original meaning. Indeed there is very little consent as to what meaning the term should be given today.

This is not the place for a discussion of the typology concept. It is sufficient to note that the existence of a continuous development in artifact forms over time is an empirical fact that relates to human societies in general. And further that it is true - with qualifications - for the composition of artifact assemblages also as laid down by human societies. The continuous development is not in itself a 'universal law' since discontinuous breaks may occasionally occur as an outcome of ordinary cultural processes within the societies. Yet, the continuous development that we observe in artifact forms and assemblages must have a background in universal mechanisms inherent to human societies, and more basically in the behaviour patterns of human beings.

No matter what kind of explanation one may offer for these regularities, they are of great practical importance to the archaeologist in his efforts to exert time control over his material. It is these practical aspects that are the issue of the following.

The basic criteria for continuity in terms of qualitative variables were discussed by Malmer (1963). He operated with two types of elements or variables in his criteria. 'Constant elements', were variables with only two states - either present or absent. Those are what I would prefer to call dichotomous, nominal variables. The other type he named 'variable elements', and from his example (degrees of coarseness in an ornament type) it is clear that he was referring to ordinal variables. He does not seem to have considered the type nominal, alternative variables.

Malmer suggested two criteria of continuity based on the two element types that he separated. The first criterion was: continuity is present if in a series of artifacts, constant elements are gradually replaced by other constant elements. In a diagram this may be shown as in Table 4.

The second criterion states: continuity is present if the 'variable elements' have their states replaced in a regular, ordered (= rank order) manner in a series of artifacts. In a diagram it takes the form

1:	A	B	C	D	E	.	.	.	.
2:	.	B	C	D	E	F	.	.	.
3:	.	.	C	D	E	F	G	.	.
4:	.	.	.	D	E	F	G	H	.
5:	.	.	.	.	E	F	G	H	I

Table 4.

1:	J1
2:	J2
3:	J3
4:	J4
5:	J5

Table 5.

1:	Wa	Xa	Ya	Za
2:	Wb	Xa	Ya	Za
3:	Wb	Xb	Ya	Za
4:	Wb	Xb	Yb	Za
5:	Wb	Xb	Yb	Zb

Table 6.

	A	B	C	D	E	F	G	H	I	W	X	Y	Z
1:	1	1	1	1	1	0	0	0	0	1	1	1	1
2:	0	1	1	1	1	1	0	0	0	0	1	1	1
3:	0	0	1	1	1	1	1	0	0	0	0	1	1
4:	0	0	0	1	1	1	1	1	0	0	0	0	1
5:	0	0	0	0	1	1	1	1	1	0	0	0	0

Table 7.

	K1	K2	K3	K4	K5
1:	1	0	0	0	0
2:	0	1	0	0	0
3:	0	0	1	0	0
4:	0	0	0	1	0
5:	0	0	0	0	1

Table 8.

	A	B	C	D	E
1:	15%	3%	16%	66%	0%
2:	17%	6%	13%	59%	5%
3:	26%	8%	10%	31%	25%
4:	19%	11%	8%	18%	44%
5:	16%	25%	3%	15%	41%

Table 9.

	A	B	C	D	E	SUM
1:	8	2	9	36	0	55
2:	19	7	14	65	5	110
3:	26	8	10	32	25	101
4:	16	9	7	15	37	84
5:	15	23	3	14	38	93

Table 10.

shown in Table 5, which is not an operational form for the type of analyses considered in this paper. One has to transform the information into a series of alternative, two-state, 'artificial' variables to utilise the ordinal scale information (Madsen 1985:12-14). Table 5 transformed into a table of alternative variables preserving the rank order information takes the form shown in Table 6.

Tables 5 and 6 may be rewritten so that the element labels are placed in the column headings, and their values are given in the table as 1 and 0. Tables 5 and 6 together would take the form of Table 7. It should be remembered that the variables W-Z are not real observed variables, but artificial variables presenting the information in the one variable J alone.

The third type of variable, the nominal, alternative variable - not treated by Malmer - could be given the same representation as in Table 5, but in a dichotomous version it would, assuming one variable (say K) with five states, look like Table 8. We can immediately see that although this type in its dichotomous version turns up with as many 'variables' as it has states, it is nevertheless one variable only. There is no, and cannot be any information on continuity in Table 8. Only combinations with other alternative dichotomous variables in a larger table can lead to information on continuity.

The dichotomising of alternative variables takes up a lot of space, as each variable state has to be transformed into a separate 'variable'. It is, however, a necessary operation, if this type of variable is to be analysed by a CA. If one has many alternative variables, it may therefore be more reasonable to use a PCO, which can handle alternative variables directly.

The above should be seen as a formalised representation of the original typological concept. It is important to note that it can be fully given a common matrix representation, which can be analysed by multivariate metric statistics.

Naturally, the typological concept is confined to artifacts, but the principles that govern it can in fact be applied to closed find associations, as demonstrated by Malmer also (1963:30-32). If we have a series of closed finds containing various artifacts that can be classified into types, then we may use Malmer's first criteria of continuity in exactly the same way as we did with the typological series. Normally, his second criteria of continuity do not enter here as long as we are dealing with discrete objects. However, we may work directly on the basic descriptive (classified) elements of the artifacts within the closed finds. Then, of course, the second criteria of continuity are applicable too.

The matrix representation for an analysis of continuity in find associations would be inseparable in structure from one stemming from a typological analysis.

0	1	0	0	1	0	1	0
0	1	0	1	1	0	1	0
0	1	0	1	1	0	1	1
1	1	0	1	0	0	1	1
1	1	0	1	0	1	0	1
1	0	1	1	0	1	0	1
1	0	1	0	0	1	0	1
1	0	1	0	0	1	0	0

Table 11.

0	1	0	0	3	0	2	0
0	2	0	1	2	0	3	0
0	3	0	2	1	0	2	1
1	2	0	3	0	0	1	2
2	1	0	2	0	1	0	3
3	0	1	1	0	2	0	2
2	0	2	0	0	3	0	1
1	0	3	0	0	2	0	0

Table 12.

1	1	1	0	0	0	0	0
1	1	1	1	0	0	0	0
1	1	1	1	1	0	0	0
0	1	1	1	1	1	0	0
0	0	1	1	1	1	1	0
0	0	0	1	1	1	1	1
0	0	0	0	1	1	1	1
0	0	0	0	0	1	1	1

Table 13.

3	2	1	0	0	0	0	0
2	3	2	1	0	0	0	0
1	2	3	2	1	0	0	0
0	1	2	3	2	1	0	0
0	0	1	2	3	2	1	0
0	0	0	1	2	3	2	1
0	0	0	0	1	2	3	2
0	0	0	0	0	1	2	3

Table 14.

In Scandinavian and European archaeology in general, the concern with problems of continuity has always focused on qualitative aspects. In American archaeology, the focus has mainly been on quantitative aspects, at least since the forties. In the frequency seriation, as the method developed here was named, the basic idea is that the relative frequency of a given type of artifact or artefact element will change continuously with time. And further, that the general pattern will be one of artifact types or element types coming into being at one point in time, growing in relative frequency, reaching a peak, diminishing in relative frequency again and finally disappearing.

Archaeologically, this means that if we have a number of find associations, sufficiently large to allow for a relative frequency representation, laid down over not too long a period of time, and preserved undisturbed, then we may sort them according to the relative frequencies of their types as seen in Table 9. Table 9, however, could stem from a matrix of counts as in Table 10, and Table 10 in turn has in principle the same characteristics as Table 7, ie. they both contain positive integers representing counts of qualities. The only difference is that the counts in Table 10 may reach any positive integer, whereas the counting method in Table 7 prevents integers other than 0 and 1. For a method of analysis like CA dealing with counts, this makes no difference at all. Both tables have a form and content which are suitable for direct input to a CA, although, of course, you cannot mix the two types of counting in the same analysis.

Thus you can turn most types of archaeological variables whether qualitative or quantitative in origin into a type of abundance table suitable for a CA.

If we turn to the PCO, we have an even larger flexibility, since we may take the different types of variables as direct input to the method. Thus the input table for PCO may well contain a mixture of: dichotomous, nominal variables noted as 1 and 0; alternative nominal variables, where each state is represented by a number; ordinal variables, which have been transformed into two-state, alternative, nominal variables; ratio scale, frequency variables, which must be given as relative frequencies.

The PCO is thus more flexible with regard to types of input data, and combinations of the various types, than is CA. But, then of course there are other features in favour of CA, as noted earlier in this paper and discussed again later.

Finally, the third method, PCA, cannot be used for this type of data at all, since the data will tend to be poisson distributed, rather than normally distributed, the latter being a requirement of the PCA.

We have seen that the criteria of continuity used in archaeology may be given a common matrix representation of positive integer counts showing the abundance of column variables (element types or artifact types) on row units (artifacts or find associations). And we have noted that the incidence type of matrix with 1's and 0's is to be considered a special case of the abundance type of matrix, only.

What then are the conditions for a given matrix to yield a perfect representation of continuity? They are: »in each individual column either, *the elements increase to a maximum and then decrease*, or *the elements increase*, or *the elements decrease*« (Kendall 1971:219) (eg. Table 12). If we take the special case of the incidence matrix, then the conditions may be expressed as: »all the 1's in each individual column have to be lumped together without any intervening 0's« (Kendall 1970:126) (e.g. Table 11).

Our interest in continuity need not be limited to the relationship between the units in the rows. Indeed we may be interested in whether there are relations of continuity among the variables in the columns as well. The conditions for this with respect to the elements within each row are exactly the same as with the units. Thus a matrix in which there is a perfect representation of continuity for both the units and the variables would take the form shown in Table 13 and 14.

The operation of interchanging rows and columns in a given data matrix in order to obtain the best possible solution with respect to the idealised matrices Table 11-14 is known in archaeology as seriation.

A vast amount of ingenuity has been invested in the development of methods that can achieve this aim (Marquardt 1978). The problem, however, is not easily solved, since there is no true 'arithmetic solution' to the permutation of rows and columns. Most solutions are iterative. That is you keep interchanging the order of rows and columns (by hand or machine) until you feel that you cannot improve the order any further. A major problem with this type of procedure is that you can always obtain some sort of order which resembles the one you aim at (e.g. Table 11-14), and it is very difficult, not to say almost impossible, to decide from a reordered data matrix whether it is sufficiently close to the ideal to allow you to conclude that the criterion of continuity is being met with. As Kendall has stated in relation to permutations of a matrix:

> As long as we work solely with permutations, the method, or any variant of it, will of necessity yield a linear ordering as an answer, and so will be given no opportunity to 'fail'. I attach great importance to methods which are capable of failure, because it is obvious that in some ill-chosen problems a method ought to fail, and thus warn us that we are taking too simple-minded a view of the data. (1971:218)

Kendall proposed the use of a method that could fail. He applied Kruskal's non-metric scaling program (MDSCAL) to ideal data of the type shown in Table 10. The result was a perfect line-up of the units in the correct order, laid out in a semi-circle - or horseshoe. When applied to real data, one may simply judge from the degree to which the units follow a semi-circle how well the criterion of continuity is met with. And naturally, one simply takes the order of the units in the semi-circle as a usable approximation to the optimal order of the row units in the data matrix.

It is exactly the same type of problem-solving with the same benefits we achieve by using either the principal coordinate analysis or the correspondence analysis as a means to seriate matrices.

In order to investigate the seriation potentials of the two methods, I used a series of 50 by 50 ideal matrices as input, some being of the general type seen in Table 13, others of the general type seen in Table 14. No matter which type of matrix was chosen, the result was the same for each of the two methods.

The first two axes of the PCO place the units in a formation very similar to a horseshoe (indeed more horseshoe-like than the formation obtained by non-metric scaling) (Figure 11). In order to understand why the layout of the units on the first two principal axes attains this arced shape, we should remember that the PCO is based on a similarity matrix between the units. The first two principal axis of the PCO can thus be considered as a two-dimensional 'mapping' of the similarity coefficients. Units with a high degree of similarity are placed close to one another, units with a lesser degree of similarity farther apart, and units with no similarity as far apart as possible, yet still so that the inter-point distance of all those units with no similarity is the same. It is the latter that create the arced lay out, because generally speaking we will have a situation where not only the similarity between the tails of the sequence, but also between the tails and the middle of the sequence will be zero. The only way to present this is through an arc, where the interdistance between the end points is approximately the same as between the end points and the middle of the sequence. This of course is a very simplified way of explaining the horseshoe. An elaborate explanation would be more complex and harder to come at (Kendall 1971:227).

When using the PCO for seriation we should note that it is only the units that are sorted, and indeed if we reorder the input matrix according to the sequence order of the first two principal axes, we get a matrix layout as in Table 11 or 12. To obtain an order of variables using the same method we would have to transpose the data matrix so that the variables 'become the units', and vice versa. However, this would be possible only if all variables were of the abundance type (including the incidence case), and then there would be no reason to use a PCO at all. It would be much better to choose a CA.

A CA performed on the same ideal matrices yields a somewhat different type of layout, closely resembling a parabola. In fact it is not a parabola, but it probably would have been, had it not been for the edge effect to be found in any input matrix. Thus Hill (1974:348) has proved mathematically »that when a single natural gradient exists in the data, the later axes may be approximate polynomials of the first« (1974:351). Hill specifically points to archaeological seriation as an issue where this feature of the CA can be of great value, and he gives his share to the seemingly endless seriation attempts on the Münsingen Rain data (1974:350-354). The result he obtains is the best I have seen on the Münsingen Rain as yet.

The parabolic formation we find in Figure 11 consists of 50 points. Yet in reality it represents 100 points, for both the units and the variables are present. Due to the absolute symmetry in the 50 by 50

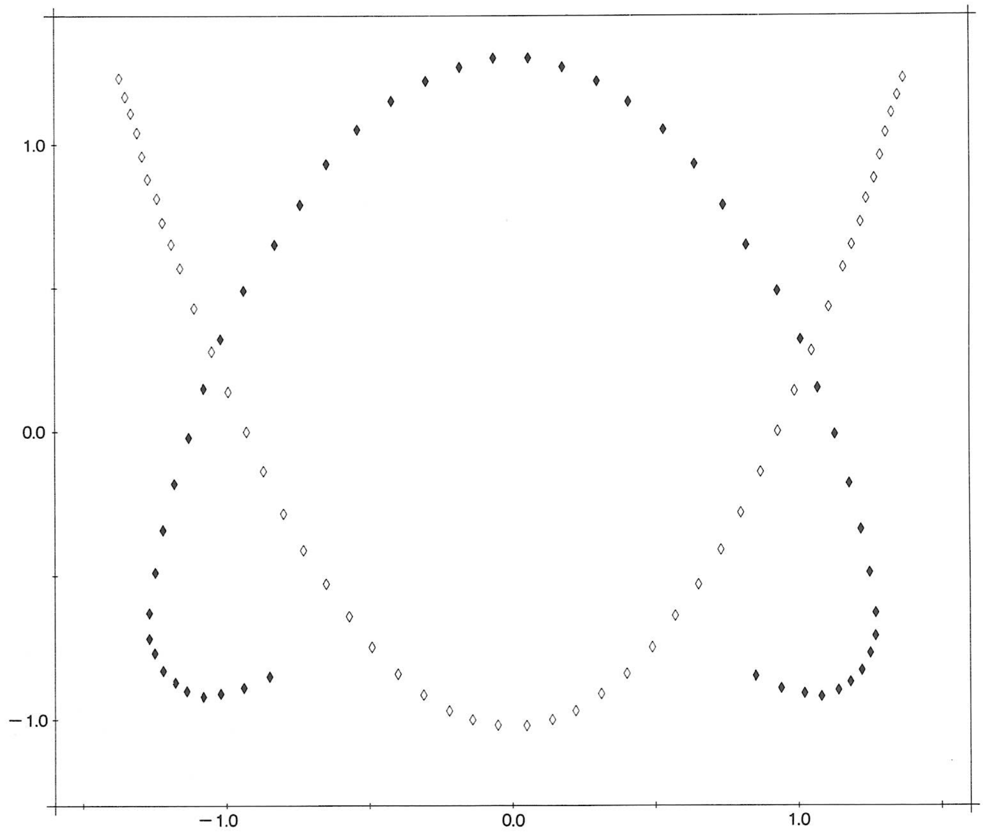

Figure 11. Plot of the first two principal axes from a PCO (heavy signatures) and a CA (open signature) of a 50 by 50 matrix containing ideal data.

matrices that have been used as input, units and variables cover each other exactly two by two in the plots of the principal axes. This, of course, is a situation that is never encountered with real data. However, we always get an ordering of both units and variables at the same time, and the matrix we receive when we use the sequence order from the Ca is of the type Table 13 and 14, where both rows and columns are sorted.

When compared to the PCO, the CA method is much to prefer. It gives a simultaneous ordering of both units and variables, with reference to the same set of principal axes. This means that not only do we get a sequence of units and variables, but when the least tendency for clustering is present (the rule rather than the exception with real data), we can immediately distinquish where the break points are in terms of both units and variables.

To see practical seriation results using the CA, we need only turn to several of the papers in this book. Thus the papers of Bech, Holm-Olsen, Højlund and Nielsen, all give very fine examples of the usefulness of the CA in seriation studies, and finer recommendation of the method than these papers can hardly be given.

Having said this, however, I feel urged to stress a point. The CA cannot work miracles. It cannot (and that is a true virtue) create a seriation if there is no reasonable degree of continuity in the input data. In that case there will be no good seriation to obtain. Further, it is imperative to stress that continuity is not something that is either in the archaeological record or not. It is something that is in the description of the data given by the archaeologist or not. If a seriation study fails to yield a proper seriation, then it is far more likely that it is the archaeologist who is at fault, than the archaeological record.

The PCO is not used in any of the seriation studies in this book, but several of the authors have in fact used it at an earlier stage. In all cases comparisons with results obtained with the CA fell out to the advantage of the latter, and consequently this method was chosen for the final analyses.

To exemplify the difference between the two methods, on a real-life material and not only on theoretical data distributions, I shall offer an example employing data that I have borrowed from Vankilde (1986). The input data matrix represents counts of 33 Early Bronze Age metal types found in 35 hoards. In Figure 12 the plot of the first two principal axes obtained from the CA is given (which was also the method chosen by Vankilde). In Figure 13, the plot of the first two principal axes from the PCO is shown.

As can be seen immediately, the result obtained from the CA is far better and more precise than those

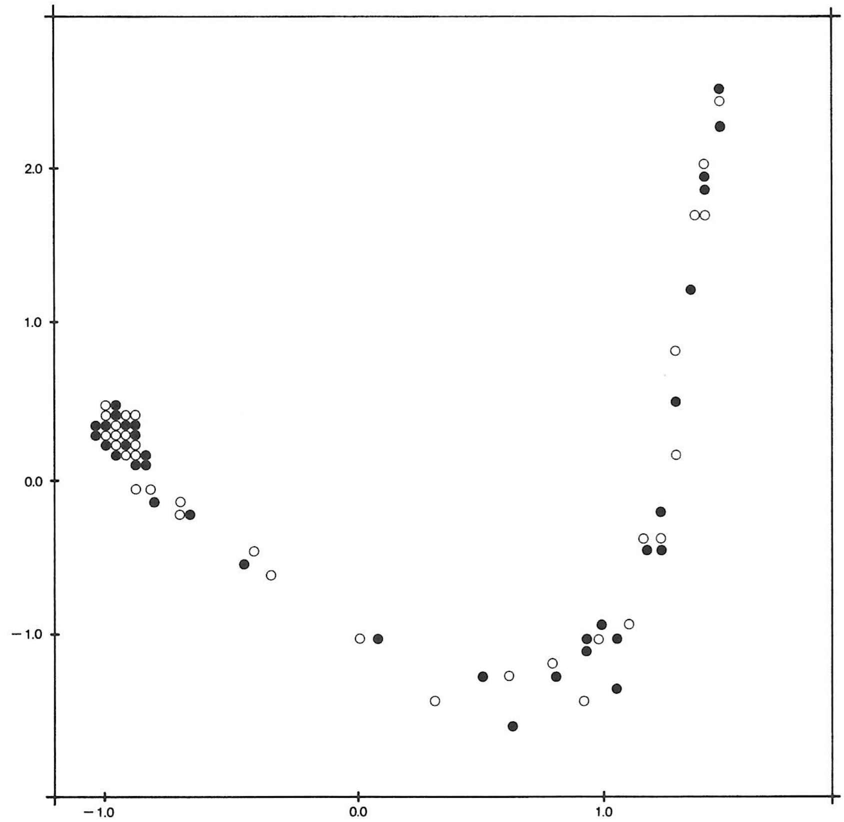

Figure 12. Plot of the first two principal axes from a CA of 35 Early Bronze Age hoards (heavy signature) described by 33 types (open signature). Data after Vankilde 1986.

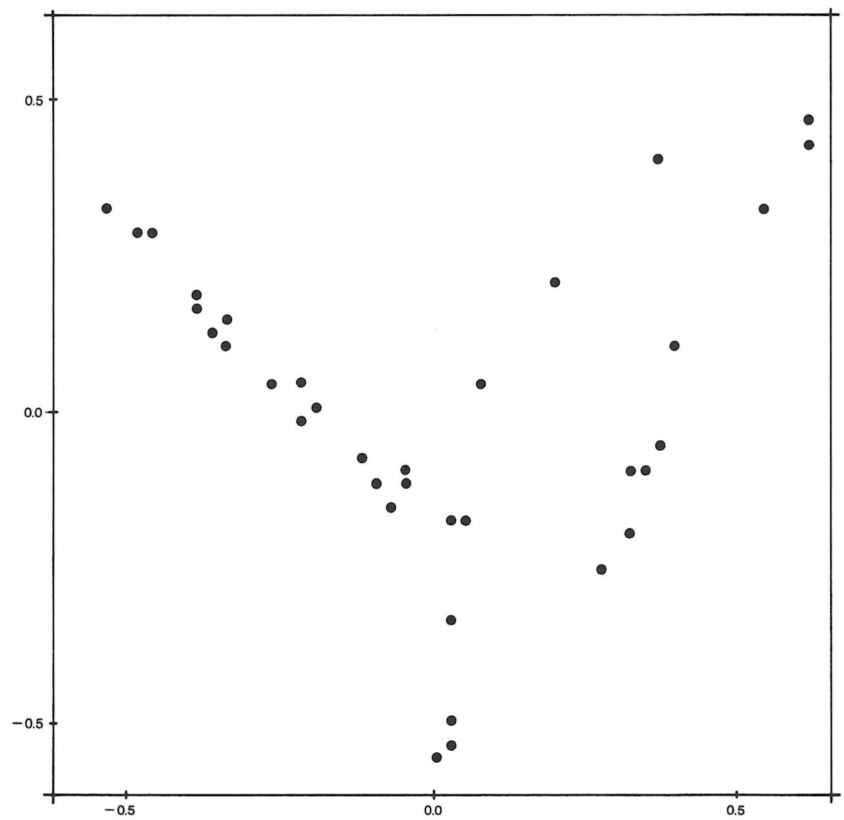

Figure 13. Plot of the first two principal axes from a PCO of 35 Early Bronze Age hoards. Data after Vankilde 1986.

obtained from the PCO. The difference between the two results is clearly due to the difference in treatment of the primary data in two analyses. The pretreatment and weighting of the data in the CA is far better than the completely equal treatment of data irrespective of numerical weight given in the PCO. Thus, whenever possible the CA should be used in preference to the PCO.

A final question should be raised. Can we be sure that a seriation resulting from a CA, a PCO, or any other method for that matter, gives us a true time sequence as a result? The answer must be a clear no. On the contrary we can be certain, time after time, to come across seriations that have nothing or very little to do with time.

As time goes by, continuity appears in the remains of material culture, and provided that we select the proper units described by the proper variables, and analyse these, then surely we get a time seriation. However, continuity in the remains of material culture may be created by many other agents than time, and although time very often is involved in the continuity we see, we must always be alert to other causes.

To see examples, we need only turn to the paper of Holm-Olsen (this volume), where the seriation created is determined by geographical factors, or to Højlund's paper (this volume), where a very fine chronological seriation is penetrated by social differences between two building areas in a most intriguing manner. The lesson to learn from these two papers is quite simple. A seriation can never be assumed to be a time seriation until this has been proved by independent means.

CONCLUDING REMARKS

This paper has covered a wide range of issues. It began with a general discussion of the archaeological research process, in order to establish a firm base for the understanding of the role of statistics in archaeology. It continued with an assesment of the usefulness of statistics in archaeology. The conclusion reached here was that inductive statistics are of little use, due to the nature of the archaeological research situation, whereas deductive statistics have a high use potentional as a means of organizing and finding structure in the primary archaeological record.

In the following section, three multivariate deductive methods were outlined. These methods are the ones used throughout this book in the various papers, and the primary purpose of the section was to give a brief introduction to the methods and to state their virtues and shortcomings in relation to archaeological data.

For a closer demonstration of the usefulness of these methods - in addition to what is demonstrated in the other papers in this book - the two final sections are devoted to two classical problems in archaeology. The first problem is that of a typological division based on morphology. The other is the time seriation problem. It is demonstrated here how multivariate statistics may help solve these problems in a more rigorous and controlled manner than otherwise possible.

I hope very much that these examples, together with those given by the other authors in this book may help to convince the reader that not only are multivariate statistics usable in archaeology, but in fact are a means of obtaining better results than can be gained by other means.

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